Ex: 8.3

1. Prove that $\sqrt{5}$ is an irrational number

Assume $\sqrt{5}$ is a rational number

So, $\sqrt{5} = \frac{p}{q}$ $(q \neq 0)$ and p, q are co - prime

Squaring both sides.

$5 = \frac{p^2}{q^2}$

$5q^2 = p^2$

p^2 is a multiple of 5

Then, p is also a multiple of 5 …(i)

So, $p = 5m$

Squaring both sides we get.

$p^2 = 25m^2$

$5q^2 = 25m^2$.

$q^2 = 5m^2$

q^2 is a multiple of 5

Then, q is also a multiple of 5 ...(ii)

From (i) and (ii), both p and q are multiples of 5.

Which means p and q are having a common factor 5

This contradicts the statement, p and q are co - prime

This implies that our assumption that √5 is a rational number.

Hence, √5 is an Irrational number, Hence Proved.

Ex: 8.3

2. Prove that $3 + 2\sqrt{5}$ is irrational number.

Assume $3 + 2\sqrt{5}$ is a rational number

So, $3 + 2\sqrt{5} = \frac{p}{q}$ $(q \neq 0)$ and p, q are co - prime

$$3 + 2\sqrt{5} = \frac{p}{q}$$

$$2\sqrt{5} = \frac{p}{q} - 3$$

$$\sqrt{5} = \frac{p-3q}{2q}$$

We know that $\sqrt{5}$ is an irrational number and $\frac{p-3q}{2q}$ is a rational number

So, it contradicts our assumption of $3 + 2\sqrt{5}$ is an irrational number

Hence, it is proved that $3 + 2\sqrt{5}$ is an irrational number

Ex: 8.3

3. (i) Prove that $\frac{1}{\sqrt{2}}$ is irrational number.

Assume $\frac{1}{\sqrt{2}}$ is a rational number

So, $\frac{1}{\sqrt{2}} = \frac{p}{q}$ $(q \neq 0)$ and p, q are co - prime

$$\frac{1}{\sqrt{2}} = \frac{p}{q}$$

$$\sqrt{2} = \frac{q}{p}$$

Squaring both sides.

$$2 = \frac{q^2}{p^2}$$

$2p^2 = q^2$

q^2 is a multiple of 2

Then, q is also a multiple of 2 ...(i)

So, q=2m

Squaring both sides we get.

$q^2 = 4m^2$

$2p^2 = 4m^2$.

$p^2 = 2m^2$

p^2 is a multiple of 2

Then, p is also a multiple of 2 ...(ii)

From (i) and (ii), both p and q are multiples of 2.
Which means p and q are having a common factor 2
This contradicts the statement, p and q are co - prime
This implies that our assumption that $\sqrt{2}$ is a rational number.

Hence, $\frac{1}{\sqrt{2}}$ is an Irrational number, Hence Proved.

Ex: 8.3

3. (ii) Prove that $7\sqrt{5}$ is irrational number.

Assume $7\sqrt{5}$ is a rational number

So, $7\sqrt{5} = \frac{p}{q}$ $(q \neq 0)$ and p, q are co - prime

$$7\sqrt{5} = \frac{p}{q}$$

$$\sqrt{5} = \frac{p}{7q}$$

We know that $\sqrt{5}$ is an irrational number and $\frac{p}{7q}$ is a rational number

So, it contradicts our assumption of $7\sqrt{5}$ is an irrational number

Hence, it is proved that $7\sqrt{5}$ is an irrational number.

Ex: 8.3

3. (iii) Prove that 6+ √2 is irrational number.

Assume 6 + √2 is a rational number

So, 6 + √2 = $\frac{p}{q}$ $(q \neq 0)$ and p, q are co - prime

$6 + \sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - 6$

$\sqrt{2} = \frac{p - 6q}{q}$

We know that √2 is an irrational number and $\frac{p - 6q}{q}$ is a rational number

So, it contradicts our assumption of 6 + √2 is an irrational number

Hence, it is proved that 6 + √2 is an irrational number.